

305, 307

S.E. Comp &amp; I.T. Sem III

CBQS Nov. 13

25/11/13

Sub - A.M. III

shilpa-2nd half-(c)13-33

Con. 7854-13.

GX-12040

(3 Hours)

[ Total Marks : 80

- N. B. :** (1) Question No. 1 is **compulsory**.  
 (2) Answer any **three** questions from Q. 2 to Q. 6.  
 (3) **Each** question carry **equal** marks.  
 (4) **Non-programmable** calculator is **allowed**.

1. (a) Find  $L^{-1} \left\{ \frac{e^{4-3s}}{(s+4)^{5/2}} \right\}$  9 5

(b) Find the constant a, b, c, d and e If. 5

$f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$  is analytic.

(c) Obtain half range Fourier cosine series for  $f(x) = \sin x$ ,  $x \in (0, \pi)$ . 5

(d) If  $r$  and  $\bar{r}$  have their usual meaning and  $a$  is constant vector, prove that 5

$$\nabla \times \left[ \frac{a \times \bar{r}}{r^n} \right] = \frac{(2-n)}{r^n} a + \frac{n(a \cdot \bar{r}) \bar{r}}{r^{n+2}}$$

2. (a) Find the analytic function  $f(z) = u + iv$  If  $3u + 2v = y^2 - x^2 + 16xy$ . 6

(b) Find the z - transform of  $\{a^{|k|}\}$  and hence find the z - transform of  $\left\{ \left( \frac{1}{2} \right)^{|k|} \right\}$  6

(c) Obtain Fourier series expansion for  $f(x) = \sqrt{1 - \cos x}$ ,  $x \in (0, 2\pi)$  and hence 8

deduce that  $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$ .

3. (a) Find :-

(i)  $L^{-1} \left\{ \frac{s}{(2s+1)^2} \right\}$  3

(ii)  $L^{-1} \left\{ \log \frac{s^2 + a^2}{\sqrt{s+b}} \right\}$  3

(b) Find the orthogonal trajectories of the family of curves  $e^{-x} \cos y + xy = \alpha$  6  
 where  $\alpha$  is the real constant in xy - plane.

[ TURN OVER

**Con. 7854-GX-12040-13.****2**

(c) Show that  $\vec{F} = \left( y e^{xy} \cos z \right) \mathbf{i} + \left( x e^{xy} \cos z \right) \mathbf{j} - \left( e^{xy} \sin z \right) \mathbf{k}$  is irrotational and **8**

find the scalar potential for  $\vec{F}$  and evaluate  $\int_c \vec{F} \cdot d\mathbf{r}$  along the curve joining the points  $(0, 0, 0)$  and  $(-1, 2, \pi)$ .

4. (a) Evaluate by Green's theorem.  $\int e^{-x} \sin y dx + e^{-x} \cos y dy$  where  $c$  is the rectangle **6**

whose vertices are  $(0, 0)$   $(\pi, 0)$   $(\pi, \frac{\pi}{2})$  and  $(0, \frac{\pi}{2})$ .

(b) Find the half range sine series for the function. **6**

$$f(x) = \frac{2kx}{l}, \quad 0 \leq x \leq \frac{l}{2}$$

$$= \frac{2k}{l}(l-x), \quad \frac{l}{2} \leq x \leq l$$

(c) Find the inverse z-transform of  $\frac{1}{(z-3)(z-2)}$  **8**

(i)  $|z| < 2$

(ii)  $2 < |z| < 3$

(iii)  $|z| > 3$ .

5. (a) Solve using Laplace transform.  $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 3y = e^{-x}$ ,  $y(0) = 1$ ,  $y'(0) = 1$ . **6**

(b) Express  $f(x) = \frac{\pi}{2} e^{-x} \cos x$  for  $x > 0$  as Fourier sine integral and show that **6**

$$\int_0^{\infty} \frac{w^3 \sin wx}{w^4 + 4} dw = \frac{\pi}{2} e^{-x} \cos x.$$

(c) Evaluate  $\iiint_s \vec{F} \cdot \mathbf{n} ds$ , where  $\vec{F} = x\mathbf{i} - y\mathbf{j} + (z^2 - 1)\mathbf{k}$  and  $s$  is the cylinder formed **8**

by the surface  $z = 0$ ,  $z = 1$ ,  $x^2 + y^2 = 4$ , using the Gauss - Divergence theorem.

**Con. 7854-GX-12040-13.**

**3**

6. (a) Find the inverse Laplace transform by using convolution theorem. 6

$$L^{-1} \left\{ \frac{s^2 + 2s + 3}{(s^2 + 2s + 5)(s^2 + 2s + 2)} \right\}.$$

- (b) Find the directional derivative of  $\phi = 4e^{2x - y + z}$  at the point  $(1, 1, -1)$  in the direction towards the point  $(-3, 5, 6)$ . 6

- (c) Find the image of the circle  $x^2 + y^2 = 1$ , under the transformation  $w = \frac{5 - 4z}{4z - 2}$ . 8

\*\*\*\*\*